Pearson Edexcel

Mark Scheme (Results)

Summer 2022

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F3 (WFM03)
Paper 01

Question Number	Scheme	Notes	Marks
1(a)	$\left(\cosh A\cosh B+\sinh A\sinh B=\right)\left(\frac{e^{A}+e^{-A}}{2}\right)\left(\frac{e^{B}+e^{-B}}{2}\right)+\left(\frac{e^{A}-e^{-A}}{2}\right)\left(\frac{e^{B}-e^{-B}}{2}\right)$ $=\frac{e^{A+B}+e^{A-B}+e^{B-A}+e^{-A-B}+e^{A+B}-e^{A-B}-e^{B-A}+e^{-A-B}}{4}$ Expresses the lhs in terms of exponentials correctly, combines terms and combines fractions with common denominator (Brackets not needed due to fraction lines)		M1
	$=\frac{2e^{A+B}+2e^{-(A+B)}}{4}=\frac{e^{A+B}+e^{-(A+B)}}{2}=\cosh(A+B)$* Fully correct proof with no errors		A1*
			(2)
(b)	$\cosh(x+\ln 2)=\cosh x\cosh(\ln 2)+\sinh x\sinh(\ln 2)$ $=\left(\frac{2+\frac{1}{2}}{2}\right)\cosh x+\left(\frac{2-\frac{1}{2}}{2}\right)\sinh x$ Applies the result from part (a) and evaluates both cosh(ln2) and sinh(ln2) **Use of (a) must be seen**		M1
	$\frac{5}{4}\cosh x+\frac{3}{4}\sinh x=5\sinh x$ $\Rightarrow\frac{5}{4}\cosh x=\frac{17}{4}\sinh x$	Collects terms and reaches $a\cosh x=b\sinh x$ oe Depends on the first M mark	dM1
	$5\cosh x=17\sinh x$ oe	Correct equation	A1
	$x=\frac{1}{2}\ln\left(\frac{1+\frac{5}{17}}{1-\frac{5}{17}}\right)$ Or $\frac{e^{2x}-1}{e^{2x}+1}=\frac{5}{17}\Rightarrow x=...$	Moves to tanh x and uses the correct logarithmic form for artanh$x$ or reverts to exponential forms and solves for x Depends on both M marks	ddM1
	$x=\frac{1}{2}\ln\left(\frac{11}{6}\right)$	Cao $\left(\text{Accept integer multiples of }\frac{11}{6}\right)$	A1
			(5)
			Total 7

Way 2			
(b)	$\cosh(x+\ln 2)=\cosh x\cosh(\ln 2)+\sinh x\sinh(\ln 2)$ $=\left(\frac{2+\frac{1}{2}}{2}\right)\cosh x+\left(\frac{2-\frac{1}{2}}{2}\right)\sinh x$ Applies the result from part (a) and evaluates both cosh(ln2) and sinh(ln2) **Use of (a) must be seen**		M1
	$\Rightarrow 5\cosh x=17\sinh x$ dM1: Collects terms and reaches an equation of form $A\cosh x=B\sinh x$ A1: Correct equation		dM1A1
	$5\left(\frac{e^x+e^{-x}}{2}\right)=17\left(\frac{e^x-e^{-x}}{2}\right)$		
	$12e^x=22e^{-x}\Rightarrow e^{2x}=\frac{22}{6}\Rightarrow x=...$	Changes to exponentials (correct forms) And solves for x	ddM1
	$x=\frac{1}{2}\ln\left(\frac{11}{6}\right)$	Cao $\left(\text{Accept integer multiples of } \frac{11}{6}\right)$	A1
Way 3			
	$\cosh(x+\ln 2)=\cosh x\cosh(\ln 2)+\sinh x\sinh(\ln 2)$ $\left(\frac{e^x+e^{-x}}{2}\right)\left(\frac{e^{\ln 2}+e^{-\ln 2}}{2}\right)+\left(\frac{e^x-e^{-x}}{2}\right)\left(\frac{e^{\ln 2}-e^{-\ln 2}}{2}\right)=5\left(\frac{e^x-e^{-x}}{2}\right)$ Applies the result from part (a) and uses the exponential forms of the hyperbolic functions. **Use of (a) must be seen**		M1
	eg $5e^x+5e^{-x}=17e^x-17e^{-x}$ oe	Evaluates $e^{\ln 2}$ and $e^{-\ln 2}$ and starts to collect terms	dM1
	$12e^{2x}=22\Rightarrow e^{2x}=\frac{11}{6}$	Correct value for e^{2x}	A1
	$x=...$	Solves for x	ddM1
	$x=\frac{1}{2}\ln\left(\frac{11}{6}\right)$	Cao $\left(\text{Accept integer multiples of } \frac{11}{6}\right)$	A1

NB: Squaring and obtaining a value for sinh*x* or cosh*x* introduces extra answers. If these extra answers are then eliminated M1A1 is available but if no attempt at elimination is made award M0A0

Question Number	Scheme	Notes	Marks
2(i)	Throughout both parts of this question do not penalise omission of $\mathrm{d}x$ or $\mathrm{d}\theta$		
	$5+4x-x^2=9-(x-2)^2$ oe	Correct completion of the square Any correct result	B1
	$\displaystyle\int \frac{1}{\sqrt{5+4x-x^2}}\,\mathrm{d}x=\int \frac{1}{\sqrt{9-(x-2)^2}}\,\mathrm{d}x=\sin^{-1}\left(\frac{x-2}{3}\right)(+c)$ M1: Obtains $k\sin^{-1}\mathrm{f}(x)$ A1: Correct integration (+ c not needed)		M1A1
			(3)
(ii)	$x=6\Rightarrow\theta=\frac{\pi}{3}$ $x=2\sqrt{3}\Rightarrow\theta=\frac{\pi}{6}$	Correct θ limits in radians	B1
	$\displaystyle\int \frac{18}{(x^2-9)^{\frac{3}{2}}}\,\mathrm{d}x=\int \frac{18\times 3\sec\theta\tan\theta}{(9\sec^2\theta-9)^{\frac{3}{2}}}\,\mathrm{d}\theta$ M1: For $\displaystyle\int \frac{18}{\left((3\sec\theta)^2-9\right)^{\frac{3}{2}}}\times\left(\text{their }\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)\mathrm{d}\theta$		M1
	$\displaystyle\int \frac{54\sec\theta\tan\theta}{(9\sec^2\theta-9)^{\frac{3}{2}}}\,\mathrm{d}\theta=54\int \frac{\sec\theta\tan\theta}{27\tan^3\theta}\,\mathrm{d}\theta=2\int \frac{\sin\theta\cos^3\theta}{\cos^2\theta\sin^3\theta}\,\mathrm{d}\theta$ $\displaystyle 2\int \frac{\cos\theta}{\sin^2\theta}\,\mathrm{d}\theta$ oe eg $\displaystyle 2\int \frac{\sec\theta}{\tan^2\theta}\,\mathrm{d}\theta$ Correct simplified integral		A1
	$\displaystyle 2\int \frac{\cos\theta}{\sin^2\theta}\,\mathrm{d}\theta=2\int \mathrm{cosec}\,\theta\cot\theta\,\mathrm{d}\theta=-2\,\mathrm{cosec}\,\theta(+c)$ Obtains $k\,\mathrm{cosec}\,\theta(+c)$		M1
	$\left[-2\,\mathrm{cosec}\,\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}=-2\,\mathrm{cosec}\frac{\pi}{3}+2\,\mathrm{cosec}\frac{\pi}{6}$	Uses changed limits correctly. **Depends on all previous method marks**.	**d**M1
	$=4-\frac{4}{3}\sqrt{3}$	Cao Allow these 2 marks if limits have been given in degrees	A1
			(6)
			Total 9

ALT	For B1 and final dM1A1 of (ii)	
	dM1: Reverse the substitution A1: Correct reversed result A1: **enter as B1 on e-PEN** Correct final answer	

Question Number	Scheme	Notes	Marks
3(a)	3	Correct value seen in (a)	B1
			(1)
(b)	$\begin{pmatrix}-2 & 5 & 0\\ 5 & 1 & -3\\ 0 & -3 & 6\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}8x\\ 8y\\ 8z\end{pmatrix}\Rightarrow \begin{matrix}-2x+5y=8x\\ 5x+y-3z=8y\\ -3y+6z=8z\end{matrix}\Rightarrow\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\ldots$ Correct method for the eigenvector (making a variable equal to 0 is not a correct method)		M1
	$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}1\\ 2\\ -3\end{pmatrix}$	Any correct eigenvector	A1
			(2)
(c)	$\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=\begin{vmatrix}-2-\lambda & 5 & 0\\ 5 & 1-\lambda & -3\\ 0 & -3 & 6-\lambda\end{vmatrix}=0$ $\Rightarrow(-2-\lambda)\left[(1-\lambda)(6-\lambda)-9\right]-5\left[5(6-\lambda)\right]=0\Rightarrow\lambda=\ldots$ NB CE is $\lambda^3-5\lambda^2-42\lambda+144=0$ but may only find the constant term		M1
	$\lambda=-6$	Correct third eigenvalue The work for these 2 marks may be seen in (a) – award them Correct third eigenvalue by a different method – send to review	A1
	$\mathbf{D}=\begin{pmatrix}3 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & -6\end{pmatrix}$	Correct **D** following through their third eigenvalue	A1ft
	$\begin{pmatrix}-2 & 5 & 0\\ 5 & 1 & -3\\ 0 & -3 & 6\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}-6x\\ -6y\\ -6z\end{pmatrix}\Rightarrow \begin{matrix}-2x+5y=-6x\\ 5x+y-3z=-6y\\ -3y+6z=-6z\end{matrix}\Rightarrow\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\left(\begin{pmatrix}-5\\ 4\\ 1\end{pmatrix}\right)$ Correct strategy for 3rd eigenvector		M1
	$\mathbf{P}=\begin{pmatrix}\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{14}} & -\frac{5}{\sqrt{42}}\\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{14}} & \frac{4}{\sqrt{42}}\\ \frac{1}{\sqrt{3}} & -\frac{3}{\sqrt{14}} & \frac{1}{\sqrt{42}}\end{pmatrix}$	Fully correct matrix consistent with their **D** May have $\frac{\sqrt{3}}{3}$ etc	A1
			(5)
			Total 8

Question Number	Scheme	Notes	Marks
4.	$y=\text{artanh}\left(\dfrac{\cos x+a}{\cos x-a}\right)$		
	$\dfrac{dy}{dx}=\dfrac{1}{1-\left(\dfrac{\cos x+a}{\cos x-a}\right)^2}\times\dfrac{(\cos x-a)\times-\sin x-(\cos x+a)\times-\sin x}{(\cos x-a)^2}$ or $\dfrac{dy}{dx}=\dfrac{1}{1-\left(\dfrac{\cos x+a}{\cos x-a}\right)^2}\times\left(-\sin x\times(\cos x-a)^{-1}+(\cos x+a)\times\sin x(\cos x-a)^{-2}\right)$ M1: Correct method for the derivative. This requires $\dfrac{1}{1-\left(\dfrac{\cos x+a}{\cos x-a}\right)^2}\times$ An attempt at the quotient (or product) rule. A1: Correct derivative in any form		M1A1
	$=\dfrac{(\cos x-a)^2}{(\cos x-a)^2-(\cos x+a)^2}\times\dfrac{2a\sin x}{(\cos x-a)^2}=\dfrac{2a\sin x}{-4a\cos x}=...$ Uses correct processing to reach $\lambda\dfrac{\sin x}{\cos x}$ or $\lambda\tan x$ **Depends on the first method mark.**		dM1
	$=-\dfrac{1}{2}\tan x$	cso	A1 **(4)**
Way 2	$y=\text{artanh}\left(\dfrac{\cos x+a}{\cos x-a}\right)\Rightarrow\tanh y=\dfrac{\cos x+a}{\cos x-a}\Rightarrow\text{sech}^2 y\dfrac{dy}{dx}=\dfrac{2a\sin x}{(\cos x-a)^2}$ Takes tanh of both sides, obtains $\text{sech}^2 y\dfrac{dy}{dx}=$ an attempt at the quotient or product rule		M1
	$\dfrac{dy}{dx}=\dfrac{1}{1-\left(\dfrac{\cos x+a}{\cos x-a}\right)^2}\times\dfrac{2a\sin x}{(\cos x-a)^2}$ Correct derivative in any form		A1
	$=\dfrac{(\cos x-a)^2}{(\cos x-a)^2-(\cos x+a)^2}\times\dfrac{2a\sin x}{(\cos x-a)^2}=\dfrac{2a\sin x}{-4a\cos x}=...$ Uses correct processing to reach $\lambda\dfrac{\sin x}{\cos x}$ or $\lambda\tan x$ **Depends on the first method mark.**		dM1
	$=-\dfrac{1}{2}\tan x$	cso	A1 **(4)**

Way 3	Uses substitution $u=\frac{\cos x+a}{\cos x-a}$, obtains $\frac{\mathrm{d}u}{\mathrm{d}x}\left(=\frac{2a\sin x}{(\cos x-a)^2}\right)$ by quotient rule and $\frac{\mathrm{d}y}{\mathrm{d}u}\left(=\frac{1}{1-u^2}\right)$ followed by chain rule to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1-\left(\frac{\cos x+a}{\cos x-a}\right)^2}\times\frac{2a\sin x}{(\cos x-a)^2}$		M1
	Correct derivative in any form		A1
	Uses correct processing to reach $\lambda\frac{\sin x}{\cos x}$ or $\lambda\tan x$ **Depends on the first method mark.**		dM1
	$=-\frac{1}{2}\tan x$	cso	A1 **(4)**
			Total 4
Way 4	$y=\frac{1}{2}\ln\left(\frac{1+\frac{\cos x+a}{\cos x-a}}{1-\frac{\cos x+a}{\cos x-a}}\right)=\frac{1}{2}\ln\left(-\frac{\cos x}{a}\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\times\frac{1}{-\frac{\cos x}{a}}\times\left(\frac{\sin x}{a}\right)$	M1: Converts to correct ln form and uses chain rule to differentiate A1: Correct derivative in any form	M1A1
	Uses correct processing to reach $\lambda\frac{\sin x}{\cos x}$ or $\lambda\tan x$ **Depends on the first method mark.**		dM1
	$=-\frac{1}{2}\tan x$	cso	A1
			(4)

Question Number	Scheme	Notes	Marks
5	$x = 4e^{\frac{1}{2}t}, \quad y = e^t - t \qquad 0 \leqslant t \leqslant 4$		
	$\frac{dx}{dt} = 2e^{\frac{1}{2}t}, \quad \frac{dy}{dt} = e^t - 1$	Correct derivatives	B1
	NB: Allow missing d*t* in the following integration work		
	$S = (2\pi)\int y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,(dt) = (2\pi)\int (e^t - t)\sqrt{\left(4e^{\frac{1}{2}t}\right)^2 + \left(e^t - t\right)^2}\,(dt)$ $\left(= (2\pi)\int (e^t - t)\sqrt{4e^t + e^{2t} - 2e^t + 1}\,(dt)\right)$ Applies the surface area formula with or w/o the 2π		M1
	$= (2\pi)\int (e^t - t)(e^t + 1)(dt)$	Correct simplified integral Brackets must be present unless implied by subsequent work but award by implication if $(2\pi)\int (e^{2t} + e^t - te^t - t)(dt)$ is seen	A1
	$= (2\pi)\int (e^t - t)(e^t + 1)(dt) = (2\pi)\int (e^{2t} + e^t - te^t - t)(dt)$ $= (2\pi)\left[\frac{1}{2}e^{2t} + e^t - te^t + e^t - \frac{1}{2}t^2\right]$ B1: For $\int te^t\,dt = te^t - e^t\,(+c)$ A1: Fully correct integration (the integration may be shown as 2 separate parts and score B1A1 if both parts correct)		B1A1
	$= 2\pi\left[\frac{1}{2}e^{2t} + 2e^t - te^t - \frac{1}{2}t^2\right]_0^4 = 2\pi\left\{\left(\frac{1}{2}e^8 + 2e^4 - 4e^4 - 8\right) - \left(\frac{1}{2} + 2\right)\right\}$ Applies the limits 0 and 4 Must include 2π now. If 2 integrals have been used limits must be applied to both and the results added Depends on the first M mark (and some valid integration)		dM1
	$\pi\left(e^8 - 4e^4 - 21\right)$	Cao	A1
			(7)
			Total 7

Question Number	Scheme	Notes	Marks
6(a)	$\mathbf{A}=\begin{pmatrix} x & 1 & 3 \\ 2 & 4 & x \\ -4 & -2 & -1 \end{pmatrix}$		
	NB: Work for (a) can only be awarded in (a)		
	$\lvert\mathbf{A}\rvert = x(-4+2x)-(-2+4x)+3(-4+16)$	Correct determinant attempt (expand by any row or column) or use the Rule of Sarrus (send to review if unsure) Sign errors allowed **only within the brackets**	M1
	$=2x^2-8x+38$	Correct simplified determinant	A1
	$2x^2-8x+38=2(x-2)^2+30$ or $\frac{\mathrm{d}}{\mathrm{d}x}(2x^2-8x+38)=4x-8=0\Rightarrow x=2$ $\Rightarrow 2x^2-8x+38=...$ or $b^2-4ac=64-4\times2\times38=...$	Starts the process of showing det $\mathbf{A}\neq 0$ E.g. Completes the square, finds the minimum point or finds discriminant May find discriminant of $x^2-4x+19=...$	M1
	$2x^2-8x+38 \geqslant 30$ or $b^2-4ac<0$ Therefore det $\mathbf{A}\neq 0$ which means $\mathbf{A}$ is non-singular	Appropriate reasoning for their chosen method and a conclusion stating that $\mathbf{A}$ is non-singular. **All 3 previous marks needed** (No need to evaluate a discriminant, so ISW slips in calculation provided $64-4\times2\times38=...$or $16-4\times19=...$ seen	A1cso
			(4)
(b)	$\begin{pmatrix} x & 1 & 3 \\ 2 & 4 & x \\ -4 & -2 & -1 \end{pmatrix}\rightarrow\begin{pmatrix} -4+2x & -2+4x & -4+16 \\ -1+6 & -x+12 & -2x+4 \\ x-12 & x^2-6 & 4x-2 \end{pmatrix}\rightarrow\begin{pmatrix} -4+2x & 2-4x & 12 \\ -5 & -x+12 & 2x-4 \\ x-12 & -x^2+6 & 4x-2 \end{pmatrix}$ M1: Applies the correct method to reach at least a matrix of cofactors 2 correct rows or 2 correct columns needed A1: Correct cofactor matrix		M1A1
	$\begin{pmatrix} -4+2x & 2-4x & 12 \\ -5 & -x+12 & 2x-4 \\ x-12 & -x^2+6 & 4x-2 \end{pmatrix}\rightarrow\begin{pmatrix} -4+2x & -5 & x-12 \\ 2-4x & -x+12 & -x^2+6 \\ 12 & 2x-4 & 4x-2 \end{pmatrix}$ $\mathbf{A}^{-1}=\frac{1}{2x^2-8x+38}\begin{pmatrix} -4+2x & -5 & x-12 \\ 2-4x & -x+12 & -x^2+6 \\ 12 & 2x-4 & 4x-2 \end{pmatrix}$ dM1: Transposes and divides by their determinant.		dM1A1

	If their original determinant has been divided by 2 (acceptable for (a)) and then used here it is **not** their determinant and so scores dM0 2 correct rows or 2 correct columns needed from their previous matrix **Depends on previous method mark**. A1: Correct matrix		
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
7.	$I_n = \int \frac{x^n}{\sqrt{10-x^2}}\,\mathrm{d}x \quad n \in \mathbb{N},\ \lvert x\rvert < \sqrt{10}$		
(a)	$I_n = \int \frac{x^n}{\sqrt{10-x^2}}\,\mathrm{d}x = \int \frac{x^{n-1}\times x}{\sqrt{10-x^2}}\,\mathrm{d}x$	Writes x^n as $x \times x^{n-1}$	M1
	$\int \frac{x^{n-1}\times x}{\sqrt{10-x^2}}\,\mathrm{d}x = -x^{n-1}\left(10-x^2\right)^{\frac{1}{2}} + (n-1)\int x^{n-2}\left(10-x^2\right)^{\frac{1}{2}}\,\mathrm{d}x$ **d**M1: Uses integration by parts to obtain $\int \frac{x^{n-1}\times x}{\sqrt{10-x^2}}\,\mathrm{d}x = \alpha x^{n-1}\left(10-x^2\right)^{\frac{1}{2}} + \beta\int x^{n-2}\left(10-x^2\right)^{\frac{1}{2}}\,\mathrm{d}x$ A1: Correct expression		**d**M1A1
	$= ... + (n-1)\int x^{n-2}\left(10-x^2\right)\left(10-x^2\right)^{-\frac{1}{2}}\,\mathrm{d}x$ $= ... + 10(n-1)\int x^{n-2}\left(10-x^2\right)^{-\frac{1}{2}}\,\mathrm{d}x - (n-1)\int x^{n}\left(10-x^2\right)^{-\frac{1}{2}}\,\mathrm{d}x$ Applies $\left(10-x^2\right)^{\frac{1}{2}} = \left(10-x^2\right)\left(10-x^2\right)^{-\frac{1}{2}}$ and splits into 2 integrals		**d**M1
	$= ... + 10(n-1)I_{n-2} - (n-1)I_n \Rightarrow nI_n$	Introduces I_{n-2} and I_n and makes progress to the given result	**d**M1
	$nI_n = 10(n-1)I_{n-2} - x^{n-1}\left(10-x^2\right)^{\frac{1}{2}}$ * Fully correct proof with no errors (recovery of missing brackets counts as an error) as does missing dx		A1*
			(6)
(b)	$I_1 = \int_0^1 \frac{x}{\sqrt{10-x^2}}\,\mathrm{d}x = \left[-\left(10-x^2\right)^{\frac{1}{2}}\right]_0^1 \left(= -3+\sqrt{10}\right)$ Correct method for I_1 Limits can be substituted later		M1
	$5I_5 = 10\times 4I_3 + ...$	Applies the reduction formula at least once Allow with 3 or $\left[-x^4\left(10-x^2\right)^{\frac{1}{2}}\right]_0^1$	M1
	$I_5 = 8I_3 - \frac{3}{5} = 8\left(\frac{20}{3}I_1 - 1\right) - \frac{3}{5} = \frac{160}{3}I_1 - \frac{43}{5}$ $I_5 = \frac{160}{3}\left(\sqrt{10}-3\right) - \frac{43}{5}$ Completes the process using their I_1 to obtain a numerical value for I_5 Limits must now be substituted		M1
	$= \frac{1}{15}\left(800\sqrt{10} - 2529\right)$	Cao	A1
			(4)
			Total 10

Question Number	Scheme	Notes	Marks
8(a)	$(\mathbf{r}=)\begin{pmatrix}-4\\-5\\3\end{pmatrix}+t\begin{pmatrix}3\\4\\-1\end{pmatrix}$	Forms the parametric form of the line	M1
	$3(3t-4)+4(4t-5)-(3-t)=17$ $\Rightarrow t=(2)$	Substitutes the parametric form for the line into the plane equation and solves for "*t*". **Depends on the first mark.**	dM1
	$\begin{pmatrix}-4\\-5\\3\end{pmatrix}+"2"\begin{pmatrix}3\\4\\-1\end{pmatrix}$	Uses their value of *t* correctly to find *Q*. **Depends on the previous mark.**	dM1
	$(2, 3, 1)$	Correct coordinates Accept if written as a column vector but not with **i, j, k**	A1 **(4)**
Way 2	$\frac{x+4}{3}=\frac{y+5}{4}=\frac{z-3}{-1}$ eg $x=\mathrm{f}(y)$ $z=\mathrm{g}(y)$	Forms the Cartesian equation of the line, rearranges twice to get 2 of *x*, *y*, *z* as functions of the third	M1
		Substitutes these into the plane equation and solves for one coordinate	dM1
		Obtains the other 2 coordinates	dM1
	$(2, 3, 1)$	Correct coordinates Accept if written as a column vector but not with **i, j, k**	A1
			(4)
(b)	$\mathbf{PQ}=\begin{pmatrix}2+4\\3+5\\1-3\end{pmatrix}$, $\mathbf{PR}=\begin{pmatrix}-1+4\\6+5\\4-3\end{pmatrix}$, $\mathbf{RQ}=\begin{pmatrix}2+1\\3-6\\1-4\end{pmatrix}$	Attempts 2 vectors in plane *PQR* (Must use the given coordinates of *P*, *R* and their coordinates of *Q*	M1
	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\6&8&-2\\3&11&1\end{vmatrix}=\begin{pmatrix}30\\-12\\42\end{pmatrix}$	Attempt vector product between 2 vectors in *PQR*. **Depends on the first mark.**	dM1
	$\begin{pmatrix}5\\-2\\7\end{pmatrix}\bullet\begin{pmatrix}2\\3\\1\end{pmatrix}=11$	Uses any of *P*, *Q* or *R* to find constant. **Depends on the previous mark.**	dM1
	$5x-2y+7z=11$	Any correct Cartesian equation	A1
			(4)

Way 2	$-4a-5b-3c=1$ $2a+3b+c=1$ $-a+6b+4c=1$	Uses the Cartesian form of the equation of a plane, $ax+by+cz=1$, and substitutes the coordinates of each of the 3 points	M1
	Solves to get a value for any of *a*, *b* or *c*		dM1
	Obtains values for the other 2		dM1
	$\frac{5}{11}x-\frac{2}{11}y+\frac{7}{11}z=1$	Any correct Cartesian equation	A1
			(4)

(c)	Reflection of P in Π is $\begin{pmatrix}-4\\-5\\3\end{pmatrix}+2\times\text{"}2\text{"}\begin{pmatrix}3\\4\\-1\end{pmatrix}\left(=\begin{pmatrix}8\\11\\-1\end{pmatrix}\right)$	Correct strategy for another point on l_3	M1
	$\begin{pmatrix}8\\11\\-1\end{pmatrix}-\begin{pmatrix}-1\\6\\4\end{pmatrix}\left(=\begin{pmatrix}9\\5\\-5\end{pmatrix}\right)$	Attempts direction of l_3. **Depends on the first mark.**	dM1
	$\mathbf{r}=\begin{pmatrix}-1\\6\\4\end{pmatrix}+\lambda\begin{pmatrix}9\\5\\-5\end{pmatrix}$	Forms the equation of l_3 using R (or their reflected P) and their direction. **Depends on the previous mark.**	ddM1
		Any correct equation in vector form	A1 **(4)**
			Total 12

Question Number	Scheme	Notes	Marks
9	$\frac{x^2}{9}+\frac{y^2}{4}=1, \quad y=kx-3$		
(a)	$\frac{x^2}{9}+\frac{(kx-3)^2}{4}=1 \left(\text{or } \frac{x^2}{9}+\frac{k^2x^2-6kx+9}{4}=1\right) \Rightarrow 4x^2+9\left(k^2x^2-6kx+9\right)=36$ Substitutes to obtain a quadratic in x and eliminates fractions		M1
	$\left(9k^2+4\right)x^2-54kx+45=0$*	Correct proof with no errors	A1*
			(2)
(b)	$x=\frac{1}{2}\left(\frac{54k}{9k^2+4}\right)=\frac{27k}{9k^2+4}$ OR $x=\frac{54k\pm\sqrt{\text{discriminant}}}{2\left(9k^2+4\right)}$	Uses ½ sum of roots for the x coordinate OR Solve the equation (by formula), add the 2 roots and halve the result. Must reach x_m. Allow errors in the discriminant	M1
	$y=k\left(\frac{27k}{9k^2+4}\right)-3$ $y=\frac{27k^2-27k^2-12}{9k^2+4}=-\frac{12}{9k^2+4}$	Uses the straight line equation to find y as a single fraction, can be unsimplified Depends on first M mark of (b)	dM1
	$x=\frac{27k}{9k^2+4}, \quad y=-\frac{12}{9k^2+4}$	Fully correct work	A1
			(3)
(c)	$x^2=\frac{729k^2}{\left(9k^2+4\right)^2}\Rightarrow x^2+py^2=\frac{729k^2+144p}{\left(9k^2+4\right)^2}$ Obtains an expression for x^2+py^2 using their coordinates obtained in (b) and obtains a common denominator		M1
	$\frac{729k^2+144p}{\left(9k^2+4\right)^2}=-\frac{12q}{\left(9k^2+4\right)}\Rightarrow 729k^2+144p=-12q\left(9k^2+4\right)$ $729k^2+144p=81\left(9k^2+\frac{16}{9}p\right)$ $\Rightarrow\frac{16}{9}p=4\Rightarrow p=...$ Correct strategy to obtain a value for p or for q Depends on the first M mark of (c)		dM1
	$p=\frac{9}{4}$ or $q=-\frac{27}{4}$ oe	Correct value (or for q if found first)	A1
	$-12q=81\Rightarrow q=...$	Correct strategy to obtain a value for the second variable Depends on both previous M marks	ddM1
	$\Rightarrow x^2+\frac{9}{4}y^2=-\frac{27}{4}y$ $p=\frac{9}{4}$ and $q=-\frac{27}{4}$ oe	Both values correct – can be embedded in the equation	A1
			(5)

(c) **Way 2**	$x=\frac{27k}{9k^2+4},\quad y=-\frac{12}{9k^2+4}\Rightarrow\frac{x}{y}=-\frac{27k}{12}\Rightarrow k=-\frac{4x}{9y}$ Obtains *k* in terms of *x* and *y* using their coordinates found in (b)		M1
	$k=-\frac{4x}{9y}\Rightarrow y=-\frac{12}{9\left(\frac{16x^2}{81y^2}\right)+4}$ or $x=\frac{27\left(-\frac{4x}{9y}\right)}{9\left(\frac{16x^2}{81y^2}\right)+4}$ dM1:Substitutes *k* into *y* or *x* to obtain a Cartesian equation A1: Any correct Cartesian equation Depends on the first M mark of (c)		dM1A1
	$\Rightarrow x^2+\frac{9}{4}y^2=-\frac{27}{4}y$	Rearranges to the form required Depends on both previous M marks of (c)	ddM1
		Correct equation or correct values stated	A1
			Total 10